BULETINUL INSTITUTULUI POLITEHNIC DIN IAȘI
Publicat de
Universitatea Tehnică „Gheorghe Asachi” din Iași
Volumul 65 (69), Numărul 2, 2019
Secția
CONSTRUCȚII DE MAȘINI

YOUNG'S MODULUS MEASUREMENT METHOD USING THE SPEED OF COMPRESSIVE ELASTIC WAVES

BY

MIHĂIȚĂ HORODINCĂ[*]

“Gheorghe Asachi” Technical University of Iași, Romania,
Faculty of Machine Manufacturing and Industrial Management

Received: May 20, 2019
Accepted for publication: June 21, 2019

Abstract. The paper presents theoretical and experimental approaches related to an indirect Young’s modulus (elastic modulus) measurement method in some different materials (mild steel, pine wood and copper alloy) using the speed of longitudinal waves (or pressure waves as well) and density. The measurement of this speed (being the speed of sound as well) is done using two geophones as absolute velocity vibration sensors, placed at each end of a simple supported long beam (or a bar made from the material whose modulus should be determined). The distance and the time delay between the sensors signals, activated by the first longitudinal wave (produced by a hammer-blow impact) are involved in speed calculus. A simple setup (with a beam, two sensors, a numerical oscilloscope, a computer) and experimental technique produces confident results in comparison with classical procedures.

Keywords: Young’s modulus; measurement; elastic waves; speed; computer.

1. Introduction

The Young’s modulus known also as longitudinal elastic modulus is an essential parameter involved in the strength of the materials calculus and in structure analysis and design by finite element method. This modulus is

[*]Corresponding author; *e-mail*: horodinca@tuiasi.ro

classically described through Hooke's low as the ratio between stress and strain in elastic deformation measured in uniaxial (unidirectional) tensile strength for some materials (Czichos *et al*., 2006; Lord and Morell, 2006) or compressive strength for others (Plachy *et al*., 2009).

In this common way the Young's modulus is measured with a relative complicated testing machine (Davis, 2004) and special requirements for tensile specimens.

The bending deformation of a cantilever beam (or a simply supported beam as well) are also used in Young's modulus measurement (Miljojković *et al.*, 2017).

There are some other relative simple and non-destructive measurement methods using the frequency of vibration modes of test-pieces excited by mechanical impulses (Lord and Morell, 2006; Pintelon *et al.*, 2003; Plachy *et al.*, 2009; Se-yuen Mak *et al.*, 2000).

The impulse excitation technique were implemented in specialized equipments for industrial in-situ applications (GrindoSonic MK7, http://www.grindosonic.com/products/GrindoSonic_MK7.html).

There are many other particular techniques of measurement, especially when a specimen is not available such as: indentation methods (Prou *et al.*, 2010), Bragg wavelength shift methods (Chen *et al*., 2019), interferometric methods (Yang *et al.*, 2019) etc.

Some studies were done and some specific Young's modulus measurements methods were proposed for biological tissues (Akhtar *et al.*, 2011), thin films (Itakura *et al.*, 2013), composite materials (Cuartas *et al*., 2015), etc.

This paper introduces a simple indirect method of Young's modulus calculus based on measurement of the speed of propagation of compression waves inside a simple supported beam using a relative simple computer assisted experimental setup.

2. The Proposed Measurement Principle

The propagation speed v (and the relationship with elastic modulus E and material density ρ) of longitudinal waves (elastic waves, sound waves, pressure waves or compressive-tensile waves as well) in unidirectional (1-D) solid materials is another way for indirect and non destructive measurement technique of Young's modulus.

According with (Gopikrishna *et al*., 2016) and (https://sites.ualberta.ca/~pogosyan/teaching/PHYS_130/FALL_2010/lectures/lect15/lecture15.html), this speed is defined as a relationship between Young's modulus E and density of material ρ as it follows:

$$v = \sqrt{\frac{E}{\rho}} \quad (1)$$

And the Young's modulus results as:

$$E = \rho \cdot v^2 \quad (2)$$

If usually the density of material is known (or easy to find out by experiments), the main challenge is to determine the speed of compressive waves. Some references in the literature (Gopikrishna *et al.*, 2016; Papadakis, 1997) uses the acoustic emission technique.

Because the Young's modulus is (supposed to be) the same in compressive and tensile stress, the measurement technique proposed in this paper is based on a simple observation: a compressive wave inside a beam (of length d) produced by a hammer-blow impact at one end needs a time delay t to move to the another end. The speed is simply defined as $v=d/t$. In order to measure this time delay at each end of the beam an absolute vibration velocity (seismic) sensor is placed. When the first compressive wave created in the beam passes in the areas where each sensor is placed a local and temporary displacement is created and the sensors are activated (an electric signal is generated). The speed of compressive wave v is calculated as the ratio of the distance d_s between sensors and the time delay t_s between the moments when the sensor are activated ($v=d_s/t_s$). A computer assisted oscilloscope (who records the signals generated by the sensors) is used to measure this time delay t_s. The Young's modulus from Eq. (2) is calculated with the relation:

$$E = \rho \cdot \left(\frac{d_s}{t_s} \right)^2 \quad (3)$$

The main difficulty of this method is the accurate measurement of the time delay t_s (extremely small values, hundreds of microseconds in our experiments). This issue is simply solved using the oscilloscope's trigger and time measurement functions. Because the proposed method uses seismic vibration sensors, it is mandatory to work in a vibrationless environment.

3. Experimental Setup

Fig. 1 conceptually describes the computer assisted experimental setup. The beam specimen (a mild steel square tube) is horizontally hanged using two wires. At the both ends of the beam an absolute velocity vibration (seismic) sensor (horizontal GS-11D sensor from Geospace Technologies) is placed (rigidly fasten on the beam).

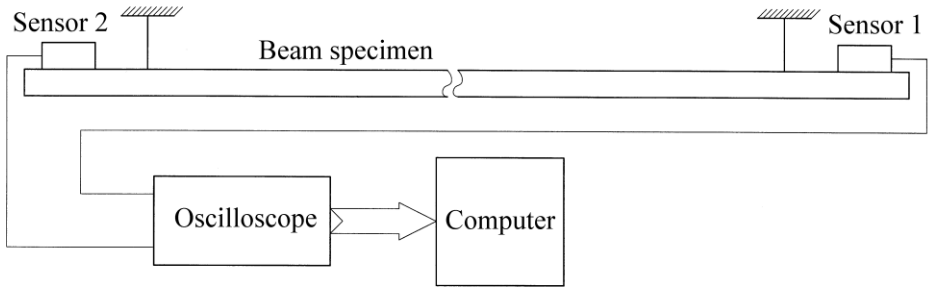

Fig. 1 – A conceptual description of the experimental setup.

The signal produced by each sensor when is activated is sent to a numerical oscilloscope (4424 PicoScope from Pico Technology) and delivered in numerical format to a computer for data processing. The compression-tensile waves in the beam are produced if one end of the beam is manually hit with a hammer (Fig. 2).

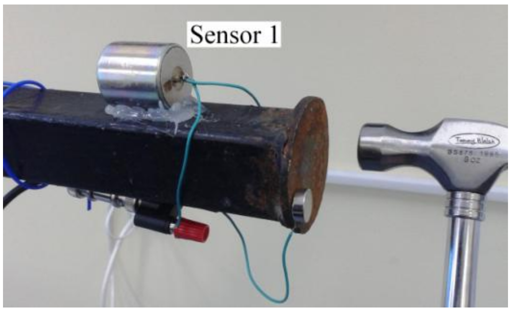

Fig. 2 – A view on the right end of the beam (with the sensor 1).

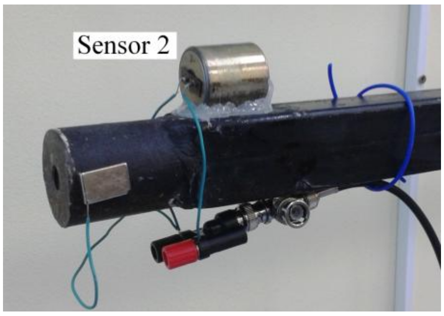

Fig. 3 – A view on the left end of the beam (with the sensor 2).

Fig. 2 presents a view of the right end of the beam with the sensor 1 (and the hammer before the impact). Fig. 3 presents a view of the left end of the beam with the sensor 2. Both sensors are temporary fasten with adhesive. There is a 3.2 m distance d_s between sensors. In order to reduce the electrical noise the sensors cases and the beam are connected to the electrical ground (the contact with the beam is assured by permanent magnets).

4. Experimental Results and Discussions

Fig. 4 presents a first experimental result with the signals delivered by both sensors after a hammer-blow impact (on the right side of the beam).

After the impact there is a free response of the beam described by both sensors. This response contains a quickly damped high frequency component (HFC in the area marked with A) and a low damped low frequency component (LFC, in the area marked with B).

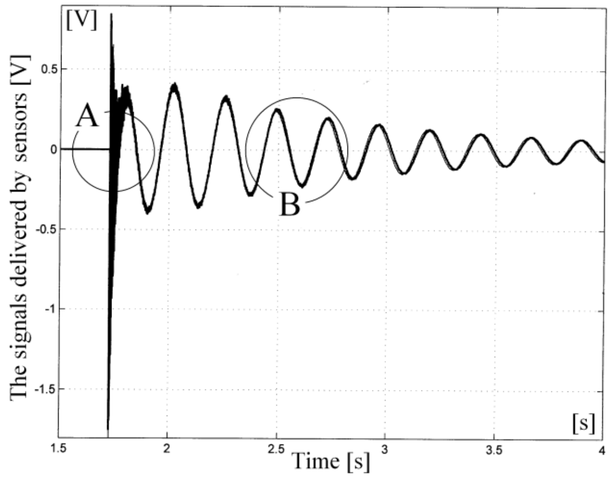

Fig. 4 – The free response of the beam after hammer-blow impact.

The conversion of the response delivered by sensor 1 from time domain into frequency domain (by fast Fourier transform, FFT) after impact for a sequence of 0.1 s in the area A is depicted in Fig. 5.

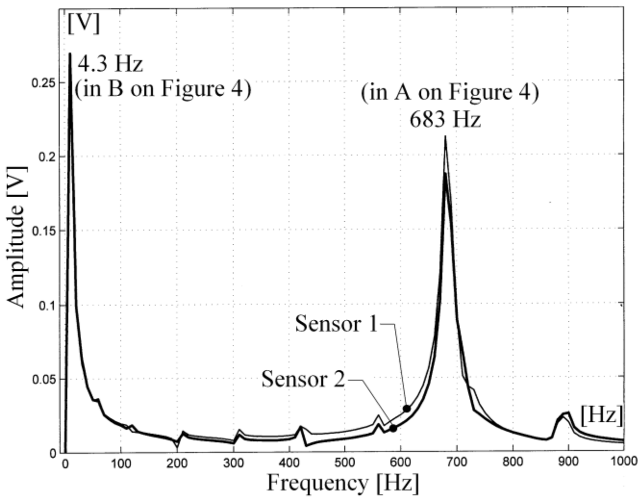

Fig. 5 – The free response in frequency domain
(for a sequence of 0.1 s in area A on Fig. 4).

On Fig. 5 occur clearly a peak related with LFC (4.3 Hz) from area B (Fig. 4) and a peak related with HFC (683 Hz) from area A. As Fig. 5 indicates both sensors generate with a good approximation the same signal (related to frequency and amplitude).

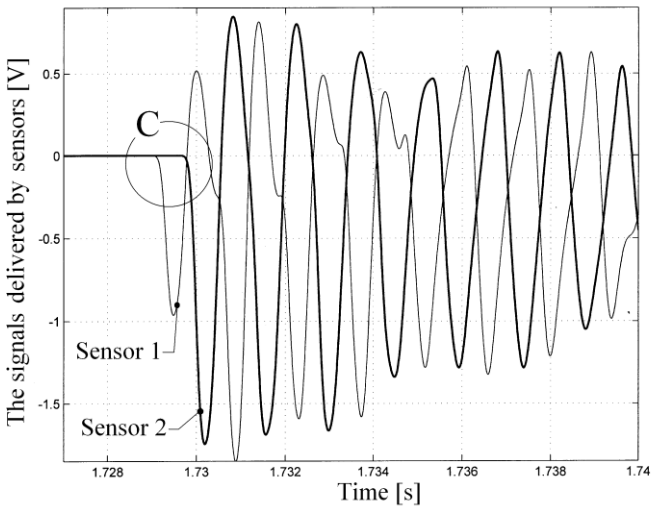

Fig. 6 – A short detail of Fig. 4 (from area A, duration of 13 ms).

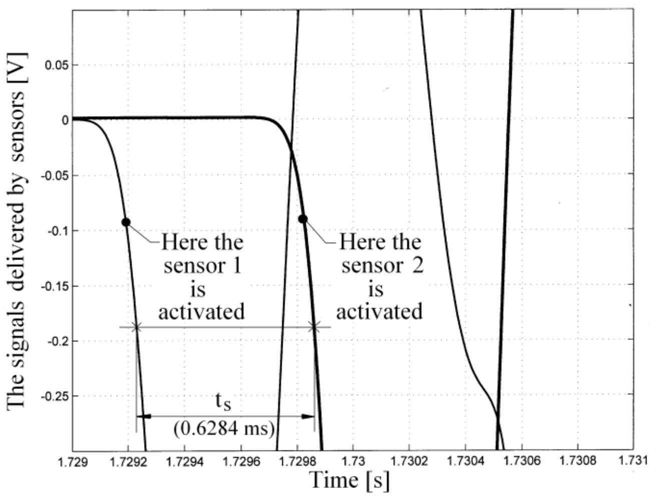

Fig. 7 – A detail of Fig. 6 (from area C). Here is revealed the time delay t_s.

It is easy to observe that LFC is related with the rigid body of the beam (as a pendulum). The explanation of the origin of HFC (depicted in Fig. 6 as a short sequence from area A on Fig. 4) is related with a free response of the beam on a longitudinal vibration mode (https://www.physics.queensu.ca/~lynann/lectures/W7L3.pdf) depicted identically by both sensors, except the phase shift (the signals are perfectly out of phase).

More interesting on Fig. 6 is the area marked with C (depicted in Fig. 7) with falling edges on both signals produced when the compressive wave activate first sensor and the second sensor. The delay is evidently the time t_s (conventionally 628.4 μs on Fig. 7) involved in the calculus of compressive-wave speed $v=d_s/t_s$ (this also being the speed of sound).

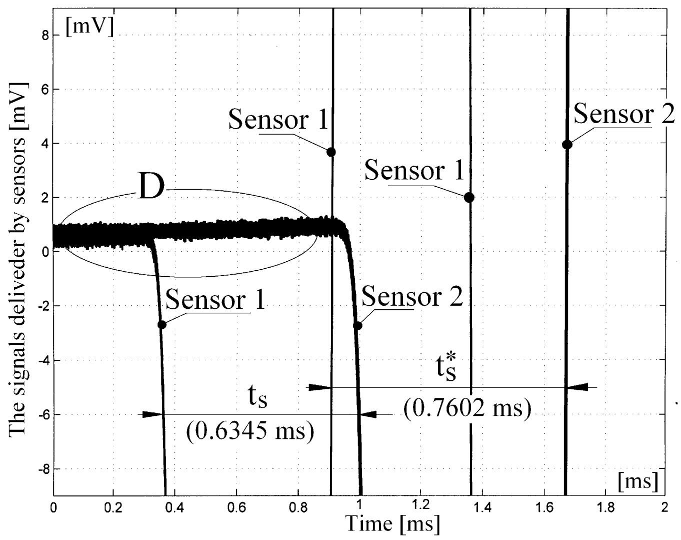

Fig. 8 – A view in the area where the sensors are activated (using triggering function of the oscilloscope).

A more accurate measurement of time delay t_s is possible if a short sequence of the signals is registered by the oscilloscope (2 ms, in comparison with 2.5 s on Fig. 4) using the trigger function (the signals recording starts when first sensor is activated) and the highest possible sensitivity (±10 mV on full scale, in comparison with –2 V÷1 V on Fig. 4). Fig. 8 describe the both signals in these circumstances. This highest sensitivity assures the highest possible slope of sensors signals when are activated. This means that the measurement of t_s time delay is less sensitive of the measurement place (here on the level of –6 mV) and less sensitive to environmental vibrations of the beam (according with the area D the average values of signals are not zero because of beam vibration). Also in D area is depicted a result of the high sensitivity (relative high measurement noise, electrical reasons) but this is not a critical item.

As it is clearly indicated on Fig. 8 the second time delay t_s^* (between the rising edges of the signals) is bigger than t_s, this is not involved in the definition of compressive wave speed. The second time delay describes the semi-period of the free vibration on HFC.

The average value of the time delay t_s on 20 different experiments is 629.38 μs (with $t_{s\ max}$ = 634.5 μs and $t_{s\ min}$ = 625.2 μs), the average value of compressive-wave speed is $v = d_s/t_s$ = 3.2 m/629.38 μs = 5080.97 m/s.

The fairness of this approach is proved by another experiment: the beam is longitudinally excited by a hammer-blow impact at the left end (in the proximity of second sensor). Now first activation of both sensors should produce rising edges on the signals, and the first activated is second sensor. This is confirmed in Fig. 9 obtained in similar conditions as Fig. 8. The measurement of time delay is done on the level of +6 mV. This is one of the worst signal records; see the level in D area. Because the environmental vibrations the level here is not zero. This is the main reason for the variability of the time delay in both experiments (according with Table 1). However this is not a critical issue since the slopes of the signals are quite identical.

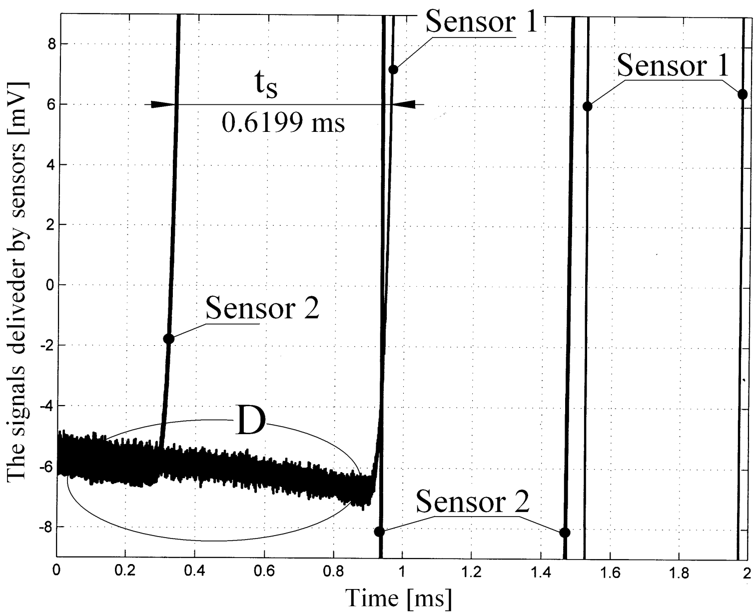

Fig. 9 – A view in the area where the sensors are activated (hammer-blow impact on the left end of the beam) in similar conditions with Fig. 8.

The average value of the time delay t_s on 20 different experiments is 621.50 μs (with $t_{s\ max}$ = 626.1 μs and $t_{s\ min}$ = 617.6 μs), the average value of compressive-wave speed is $v = d_s/t_s$ = 3.2 m/621.50 μs = 5148.83 m/s.

Table 1 presents the results of both experiments (in Eq. (3) a density of ρ = 7850 Kg/m^3 for mild steel is considered).

Table 1
A Synthesis of the Experimental Results (Mild Steel Beam)

Hammer blow impact	Average t_s [μs]	Max t_s [μs]	Min t_s [μs]	Speed v [m/s]	Average speed v [m/s]	Young's modulus E [MPa]
Right end	**629.3**	634.5	625.2	5080.97	**5114.9**	**2.0537 x 10^5**
Left end	**621.5**	626.1	617.6	5148.83		

The average time delay t_s (and the compressive-wave speed v as well) is slightly different when the excitation is produced on the left end (due to the different behaviour of the sensors). This difference indicates that the best option is to use an average time delay and speed (between two experiments) in calculus of Young's modulus. According with Table 1 the experimental value of Young's modulus measured by this procedure (E = 2.0537 x 10^5 MPa) corresponds perfectly with the value given in the literature (E = 2 ÷ 2.1 x 10^5 MPa, frequently E = 2.05 x 10^5 MPa). Surprisingly, the speed of compressive-waves (and the speed of sound as well) in literature is not a well established item for mild steel: there are values between 3600 and 5800 m/s.

The same experiments were done on a pine wood beam (the calculated density is ρ = 407.7 Kg/m^{3}and d_s = 3.89 m), with the results given in Table 2.

Table 2
A Synthesis of the Experimental Results (Pine Wood Beam)

Hammer blow impact	Average t_s [μs]	Max t_s [μs]	Min t_s [μs]	Speed v [m/s]	Average speed v [m/s]	Young's modulus E [MPa]
Right end	**774.5**	802.9	756.5	5022.59	**5049.59**	**0.10395 x 10^5**
Left end	**766.26**	792.3	747	5076.6		

The value of Young's modulus (depending by humidity) is close by those described in https://www.amesweb.info/Materials/Youngs-Modulus-of-Wood.aspx for pine wood, western white (E = 0.101 x 10^5 MPa).

According with the results from Tables 1 and 2 the speed of sound in mild steel and pine wood is almost the same.

The same experiments were done on a copper alloy beam (a circular tube, the calculated density is ρ = 8450.88 Kg/m^3 and d_s = 2.818), with the results given in Table 3.

Table 3
A Synthesis of the Experimental Results (Copper Alloy Beam)

Hammer blow impact	Average t_s [μs]	Max t_s [μs]	Min t_s [μs]	Speed v [m/s]	Average speed v [m/s]	Young's modulus E [MPa]
Right end	**749.0**	761.1	740.7	3762.34	**3778.81**	**1.2067 x 10^5**
Left end	**742.5**	758.3	730.2	3795.28		

The value of Young's modulus is close by those described in the literature (E = 1.3 x 10^5 MPa for pure copper) according with http://www.mit.edu/~6.777/matprops/copper.htm. In our experiment the exact composition of the copper alloy is unknown.

Of course the length of the beam can be significantly reduced. Many other materials can be tested with the same method (*e.g.* concrete beams those elastic modulus increases in time due to internal processes). The variation of the Young's modulus with the temperature can be also a research item.

5. Conclusions

The exact value of Young's modulus of materials is an important issue when the mechanical behaviour of a structure (stress, deformations, vibration modes etc.) is evaluated (*e.g*. using finite elements method) especially during mechanical design process (simulation).

In this paper is proved that a non-destructive proposed method of Young's modulus experimental measurement on unidirectional (1D) samples of different materials (steel, copper alloy, pine wood) produces feasible results using a simple computer assisted setup.

This method is applicable for many other type of rigid materials (concrete, glass, ceramic, plastic materials, composite materials, etc.).

There are not special requirements related by equipments and environment (except a reasonable low level of vibrations).

The originality of this method is related especially with the sensors and the signal processing. Some known theoretical approaches already proposed before (*e.g.* the speed of sound formula related to Young's modulus and density) were used.

A future approach will check in experimental terms the relationship between compressive-wave speed and frequency of first longitudinal vibration mode in a 1D solid. Probably a more precise and simpler Young's modulus measurement method will be available.

Acknowledgements. Some preliminary experimental tests were done with the help of the students Alexandru Palii and Gabriel Șerban in 2016.

REFERENCES

Akhtar R., Sherratt M.J., Cruickshank J.K., Derby B., *Characterizing the Elastic Properties of Tissues*, Materials Today, **14**, *3*, 96-105 (2011).

Chen C., Liang L., Quan W., Liao L., *Young's Modulus Measurement of Metal Wires Using FBG Sensor*, Photonic Sensors, **6**, *3*, 277-283 (2019).

Czichos H., Saito T., Smith L., *Springer Handbook of Materials Measurement Methods*, Berlin: Springer (2006).

Cuartas V.M., Perrin M., Pastor M.E., Welemane H., Cantarel A., Karama M., *Determination of the Elastic Properties in CFRP Composites: Comparison of Different Approaches Based on Tensile Tests and Ultrasonic Characterization*, Advances in Aircraft and Spacecraft Science, Techno-Press, **2**, *3*, 249-260 (2015).

Davis J.R., *Tensile Testing*, (2nd Ed.), ASM International, ISBN 978-0-87170-806-9 (2004).

Gopikrishna R., Padma Amani M., Varadanam M., *Evaluation of Elastic Modulus of Metals Using Acoustic Emission Technique*, 8th International Symposium on NDT in Aerospace, 3-5 Nov 2016, Bangalore, India (AeroNDT 2016).

Itakura A.N., Toda M., Miyake K., Förch R., Berger R., *Effective Young's Modulus Measurement of thin Film Using Micromechanical Cantilever Sensors*, Japanese Journal of Applied Physics, 52 (11R) (2013).

Lord J.D., Morell R., *Elastic Modulus Measurement*, In Measurement Good Practice Guide No. 98, National Physical Laboratory, UK (2006).

Miljojković J., Bijelić I., Vranić N., Radovanović N., Živković M., , *Determining elastic modulus of the material by measuring the deflection of the beam loaded in bending*, Tehnički vjesnik 24, 4(2017), 1227-1234(2017).

Papadakis E., *Ultrasonic Wave Measurements of Elastic Moduli E, G, and μ for Product Development and Design Calculations*, Journal of Testing and Evaluation, **26**, *3*, 240-246 (1998).

Pintelon R., Guillaume P., De Belder K., Rolain Y., *Measurement of Young's Modulus via Modal Analysis Experiments: A Sistem Identification Approach*, IFAC Proceedings Volumes, **36**, *16*, September 2003, 375-380.

Plachy T., Padevet P., Polak M., *Comparison of Two Experimental Techniques for Determination of Young's Modulus of Concrete Specimens*, Recent advances in Applied and Theoretical Mechanics, Proceedings of the 5th WSEAS International Conference on Applied and Theoretical Mechanics (MECHANICS '09), Puerto De La Cruz, Tenerife, Canary Islands, Spain December 14-16, 2009, 69-71.

Prou J., Kishimoto K., Constantinescu A., *Identification of the Young's Modulus from Indentation Testing and Inverse Analysis*, Journal of solid materials and materials engineering, **4**, *6*, 781-795 (2010).

Se-yuen Mak, Yee-kong Ng, Kam-wah Wu, *Measurement of the Speed of Sound in a Metal Rod*, Physics Education, **35**, *6*, 439-445 (2000).

Yang G., Hao X., Yu X., Wang L., Zhao J., Hong L., *Measurement of Young's Modulus of Wire by Automatic Counting Method of Interferometric Ring*, IOP Conf. Series: Materials Science and Engineering, 490, 2019, 022039.

https://www.geospace.com/wp-content/uploads/2018/10/EOL-GS11D-2.pdf (accessed at 24.03.2019).

https://sites.ualberta.ca/~pogosyan/teaching/PHYS_130/FALL_2010/lectures/lect15/lecture15.html (accessed at 27.04.2019).

https://www.picotech.com/library/experiment/wave-speed-in-a-solid (accessed at 27.04.2019).

http://www.grindosonic.com/products/GrindoSonic_MK7.html (accessed at 27.04.2019).

https://www.physics.queensu.ca/~lynann/lectures/W7L3.pdf (accessed at 27.04.2019).

https://www.amesweb.info/Materials/Youngs-Modulus-of-Wood.aspx (accessed at 27.04.2019).

http://www.mit.edu/~6.777/matprops/copper.htm (accessed at 27.03.2019).

METODĂ DE MĂSURARE A MODULULUI LUI YOUNG PE BAZA VITEZEI UNDELOR ELASTICE COMPRESIVE

(Rezumat)

Cunoaşterea exactă a valorii modulului lui Young (modul de elasticitate longitudinal) pentru diferite materiale este indispensabilă activităţilor de proiectare/simulare a structurilor (calculul sforturilor, deformaţiilor, descrierea modurilor de vibraţie etc.).

Lucrarea propune o metodă nedistructivă de determinare experimentală indirectă modulului lui Young în solide bazată pe determinarea vitezei de propagare a undelor elastice compresive în solide unidimensionale şi pe valoarea densităţii. Sunt valorificate o serie de elemente cunoscute din literatură, se propune un stand experimental simplu (o bară suspendată orizontal, doi senzori, un osciloscop numeric şi un calculator) şi se introduc o serie de elemente cu caracter de noutate cum ar fi: utilizarea senzorilor electrodinamici de tip generator în detectarea undelor elastice longitudinale şi utilizarea funcţiei de prelevare declanşată a semnalelor experimentale de către osciloscopul numeric. Metoda de măsurare a fost experimentată pe trei tipuri diferite de materiale (oţel carbon, aliaj de cupru, lemn de brad), rezultatele fiind comparabile cu cele din literatura de specialitate. Metoda poate fi aplicată oricărui tip de material solid rigid utilizabil în practică.

În viitor se intenţionează verificarea şi valorificarea experimentală a relaţiei dintre frecvenţa primului mod longitudinal de vibraţie din bară şi viteza de propagare a undelor elastice compresive (viteza sunetului) în sinteza unei metode mai precise şi mai simple de determinare a modulului de elasticitate longitudinal.